User-optimal Storage with Rising Energy Prices

Rozanna N. Jesudasan, Lachlan L. H. Andrew, Hai L. Vu
Centre for Advanced Internet Architectures, Technical Report 120906A
Swinburne University of Technology
Melbourne, Australia
rjesudasan@swin.edu.au, landrew@swin.edu.au , hvu@swin.edu.au

***Abstract*—When energy prices fluctuate, small scale storage, such as provided by electric vehicles or uninterruptible power supplies, allows users to reduce their overall costs by buying in price troughs. However, if future prices are rising, a user may be inclined to store energy long-term. By analysing the structural properties of the charging schedule for energy storage systems,this paper demonstrates that long-term storage is optimal only if prices are rising extremely rapidly. The tools and methods described in this report is part of a broader investigation into the application of data networking concepts in energy networks.**

I. Introduction

The smart grid harnesses information technology to improve the efficiency with which the electricity grid operates. Central to this is the ability to balance supply and demand using small scale storage. This energy storage affects two groups of stakeholders: end users and utilities who operate generation, transmission and distribution facilities. Utilities benefit from the use of distributed storage to reduce the need for expensive peaking generators or upgrades to the peak capacity of the transmission and distribution system. If retail prices vary, either with time or with demand, the users benefit from the use of storage to shift their demand from times of high price to low price. In this paper, we investigate energy storage decisions made by end users, and the impact that these decisions have on the utilities.

With peak-oil upon us, and increasing measures to charge for the pollution caused by coal use, the price of electricity is set to rise in real terms. In this context, a user might choose to buy energy early and save it for a long time to use when prices are high. This long-term use of storage conflicts with the use of storage for smoothing short term peaks of demand. We find that the faster the growth of prices is, the less benefit utilities obtain from storage that is controlled by the user.

In the literature the energy storage problem has been studied extensively for both the end-users (demand-side) [1], [2], [3], [4], [5] and utilities (supply-side) [6], [7].

Generally, an optimization problem is formed to investigate various aspects of the electricity scheduling and/or designing storage/battery (component sizing) using different objectives for both sides of the energy system. The problem can be then solved using a dynamic programming method over either finite or infinite time horizon. In defining the optimization problem, the existing literature can be further classified into work that either deal with deterministic or time-varying deterministic quantities in energy demand and generation (i.e. price), or stochastic quantities.

In particular, Daryanian et al. [1] and Chandy et al. [2] studied the policies for storage charging that balance the demand and supply assuming deterministic energy price and full knowledge of the demand schedule. In the former, the price is a linear function of the generation production, while in the latter it is a time-varying quadratic function. It is also interesting to note that some interesting properties of the optimal policy had been observed in Chandy et al. [2] as a result of the introduction of the battery cost as a function of its energy level. Faghih et al. [4] studied a similar problem but the price is set to be a stochastic Markov process driven by random variable with a known time-varying distribution. An additional constraint on a discharging rate of the battery is also included. In this setting Faghih et al. found that the price elasticity of demand is increases as the storage increases. All of the above results are based on the optimal policy solving over a finite time horizon that can be implemented in finite steps. It is much more challenging though when solving the problem over a infinite time-horizon which is often required for longer planning and policy developing of the future smart grid.

Examples of using an infinite time horizon to solve this optimization problem are Harsha et al. [3], Van de Ven et al. [5] and Koutsopoulos et al. [8] where a discount factor is used to calculate the future cost.In particular [8] discusses the performance of the optimal policy for increasing storage capacity, and gives examples to shows that as the capacity increases the battery never discharges and that the average demand equals the total instantaneous load in the grid.However, as the energy price would rise in real terms, techniques based on infinite time-horizon with amortized cost are not applicable. It motivates us to look at the problem over infinite time horizon with a discount factor greater than one.

The contributions of this paper are follows. In Section II, we formulate a simple infinite-horizon optimization problem to study the user-optimal energy storage. Since the total cost of this problem is infinite, we define a limiting solution concept, and show that it is a meaningful measure. More importantly, we show that a discharging policy will often have "renewal points", such that the optimal policy before the renewal point is independent of increases in energy beyond that point. This gives the important insight that our infinite horizon optimization problem can often be solved using predictions for a finite time into the future. In Section III, we numerically investigate the effect of rising prices on optimal policies, and show that moderate price increases do not reduce the effectiveness of batteries at peak shaving.

II. Model

Consider a user with a time-varying demand $d(t) \geq 0$ for electricity, and a battery of capacity $B > 0$. At each time t, the demand is met by generating an amount of electricity $g(t)$ and obtaining the rest from the battery. Generating g units of electricity at time t has a cost $C(g;t)$, for some time-varying cost function C that is increasing and strictly convex in its first argument. The convexity of the cost reflects the fact that higher levels of demand often require the utility to use more expensive peaking generators, and place greater strain on the transmission and distribution network.

Our objective is to find the generating schedule to minimize the user's long term cost in the presence of rising electricity prices (that is, C has an increasing trend in its second argument). This is naturally formulated as the discrete time, infinite horizon problem

$$\min_g \sum_{t=1}^{\infty} C(g(t);t) \tag{1}$$

subject to, for all t,

$$b(t) - b(t-1) + d(t) - g(t) = 0 \tag{2}$$
$$g(t) \geq 0 \tag{3}$$
$$b(t) \geq 0 \tag{4}$$
$$B - b(t) \geq 0 \tag{5}$$

However, the infinite sum (1) is typically infinite since the electricity price is rising, and so an alternative formulation is required. If the rate of increase is simply geometric, then it could be treated as being subject to a "discount" factor that is greater than 1 [9]; in such cases, the objective (1) can be replaced by a suitable weighted limit. However, we are interested in arbitrary rates of increase, where such a technique does not apply. Instead, we consider the limit of a sequence of finite horizon problems of the form

$$\min_g \sum_{t=1}^{T} C(g(t);t) \tag{6}$$

subject to

$$b(t) - b(t-1) + d(t) - g(t) = 0 \tag{7}$$
$$g(t) \geq 0 \tag{8}$$
$$b(t) \geq 0, \qquad t < T \tag{9}$$
$$B - b(t) \geq 0, \qquad t < T \tag{10}$$

with $b(0) = 0$. Let b_x^Tbe a solution to (1)–(10) subject to the additional constraint

$$b(T) - x = 0. \tag{11}$$

If $\lim_{T\to\infty} b_x^T(t)$ exists and is independent of x, then we can define the optimal battery occupancy to be

$$b^*(t) = \lim_{T\to\infty} b_x^T(t). \tag{12}$$

Since $b(\cdot)$ uniquely defines $g(\cdot)$, this also characterizes the optimal charging schedule. The remainder of this section establishes sufficient conditions under which this limit is well defined.

A. Existence and uniqueness of an optimal solution.

To establish that $b^*(t)$ is well defined, we introduce the following monotonicity lemmas.

Lemma 1. *For any $x, y \in [0, B]$, if $b_x^T(\tau) = b_y^T(\tau)$ for some $\tau \in [0, T]$, then $b_x^T(t) = b_y^T(t)$ for all $t \in [0, \tau]$.*

Proof: Let $A = b_x^T(t) = b_y^T(t)$. Since the costs and constraints are only coupled by (8), for all $t \in [0, \tau]$, both $b_x^T(t)$ and $b_y^T(t)$ are equal to the solution $b_A^\tau(t)$ to the problem with T replaced by τ and (11) replaced by $b(\tau)$= A. ■

The next lemma is proved in the appendix.

Lemma 2. *For any $x, y \in [0, B]$ with $x < y$ and any $\tau \in [0, T]$, if there is no such $t \in [\tau, T]$, that $b_x^T(t) = b_y^T(t)$, then $b_x^T(t) < b_y^T(t)$ for all $t \in [\tau, T]$.*

We can now state our main monotonicity result.

Theorem 3. *For any $x, y \in [0, B]$ with $x < y$, we have that $b_x^T(t) \le b_y^T(t)$ for all $t \in [0, T]$.*

Proof: Since $x < y$, (11) gives $b_x^T(T) < b_y^T(T)$. Let $\tau \in [0, T]$ be the last time that $b_x^T(\tau) = b_y^T(t\tau)$. Such a time exists since $b_x^T(0) = b_y^T(0) = 0$. It follows from lemma 1that $b_x^T(t) = b_y^T(t)$ for all $t \in [0, \tau]$. By definition there is no $t \in (\tau, T]$ such that $b_x^T(t) = b_y^T(t)$, and so lemma 2 states that $b_x^T(t) < b_y^T(t)$ for all $t \in (\tau, T]$. ■

Since the prefix on $[0, \tau]$of an optimal solution is itself optimal (i.e., $b_x^T(t) = b_y^\tau(t)$ where $y = b_x^T(\tau)$), theorem 3 implies that, for a given t, $b_0^T(t)$ is monotonic increasing in T and $b_B^T(t)$ is monotonic decreasing. This in turn implies that $\lim_{T\to\infty} b_0^T(t)$ and $\lim_{T\to\infty} b_B^T(t)$ both exist, since both sequences are bounded. If these limits are equal, then b^* in (12) is well defined. The following results, culminating in Theorem (6), give sufficient conditions for this.

Lemma 4. *If b_B^T saturates below at some time t (i.e., $b_B^T(t) = 0$), then the optimal solution to any problem b_j^T, where $j \in [0, B)$, also saturates below at time t, and $b_B^T(t') = b_j^T(t')$ for all time $t' \le t$.*

Proof: If at some time $t \in [0, T]$ we have $b_B^T(t) = 0$, then at time t, $b_j^T(t) = 0$ by Theorem 3 (i.e., due to monotonicity). Then lemma 1 implies that $b_B^T(t') = b_j^T(t')$ for all $t' \le t$. ■

Lemma 5. *If b_0^T saturates above at some time t (i.e., $b_0^T(t) = B$), then the optimal solution to any problem b_j^T, where $j \in (0, B]$, also saturates above at time t, and $b_0^T(t') = b_j^T(t')$ for all time $t' \le t$.*

Proof: If at some time $t \in [0, T]$ we have $b_0^T(t) = B$, then at that time $b_i^T(t) = B$ by Theorem 3. Then lemma 1 implies that $b_0^T(t) = b_i^T(t)$ for all $t' \le t$. ■

Theorem 6. *If $b_B^T(t) = 0$ or $b_0^T(t) = B$, for some $t \in [0, T]$, then for all $T' \ge T$, either $b_B^{T'}(t') = b_k^{T'}(t')$ or $b_0^{T'}(t') = b_k^{T'}(t')$ for all $t' \le t$*

Proof: First let us consider a time a horizon length T where the optimal solution for two problems are b_B^T and, where $b_B^T(T) > b_k^T(T)$, and $k \in [0, B)$, given that b_B^T saturates below at time t. Then from lemma (4), we know that $b_B^T(t') = b_k^T(t')$, for all $t' \le t$.

Next if we extend the horizon to some time $\tau = T'$, where $T' \ge T$, then the optimal solution for the two problems are given by $b_B^{T'}$ and $b_k^{T'}$ with end battery levels B, k respectively, and $b_B^{T'}(T') > b_k^{T'}(T')$.

Then at time T, we know that the battery levels $b_B^{T'}(T), b_k^{T'}(T) \in [0, B]$, and from lemma (4) we know that for any battery capacity if b_B^T saturates below at some time t, then the optimal solution to any problem b_j^T, where $j \in [0, B)$, also saturates below at time t, and $b_B^T(t') = b_j^T(t')$ for all time $t' \le t$.

Similarly if we consider a time horizon of length T, and the optimal solution for two problems b_0^T and b_k^T, where $b_0^T(T) < b_k^T(T)$, and $k \in (0, B]$, given that b_0^T saturates above at time t, and we extended this problem also to a time T' as above. Then the optimal solutions to the problems are in the form $b_0^{T'}$ and $b_k^{T'}$, where $b_0^{T'}(T') < b_k^{T'}(T')$ and at time $T, b_0^{T'}(T), b_k^{T'}(T) \in [0, B]$.

Then from lemma (5) we know that for any battery level at time T of $b_k^{T'}$ it is optimal to saturate above at time t. This gives $b_0^{T'}(t) = b_k^{T'}(t)$, resulting in $b_0^{T'}(t') = b_k^{T'}(t')$, for all $t' \le t$ by applying lemma1.

We should also note that at some time $\bar{t}$ where, $T \le \bar{t} < T'$. If $b_B^{T'}$ saturates below or $b_0^{T'}$ saturates above at time $\bar{t}$, then by lemma1, $b_B^{T'}(\bar{t}) = b_k^{T'}(\bar{t})$ in the case it saturates below and $b_B^{T'}(t') = b_k^{T'}(t')$, for all $t' \le \bar{t}$, or $b_0^{T'}(\bar{t}) = b_k^{T'}(\bar{t})$ in the case it saturates above and $b_0^{T'}(t') = b_k^{T'}(t')$, for all $t' \le \bar{t}$ and hence theorem (6) holds for any $t' \le t$. ■

B. Structure of optimal solutions

To investigate the structure of the optimization (1)–(5), it is useful to study the Karush-Kuhn-Tucker (KKT) conditions of the truncated version, (6)–(11).

The following derivation uses the notation of [2]. Define dual variables $\tilde{b}(t)$ for constraint (7), $\hat{\lambda}(t)$ for constraint (8), $\underline{b}(t)$ for constraint (9), $\bar{b}(t)$ for constraint (10) and $\hat{e}$ for constraint (11). For consistency with the notation $b_x^T(t)$ introduced earlier, the optimal values of these will be denoted by a superscript ^{T}; the subscript x denoting that $b(T) = x$ will be omitted where no confusion can arise.

The Lagrangian of the above optimisation problem L

is then

$$L = \sum_{t=1}^{T} C(g(t)) + \sum_{t=1}^{T} \tilde{b}(t)\left[b(t) - b(t-1) + d(t) - g(t)\right]$$
$$- \sum_{t=1}^{T} \hat{\lambda}(t)g(t) - \sum_{t=1}^{T-1} \underline{b}(t)b(t)$$
$$- \sum_{t=1}^{T-1} \bar{b}(t)(B - b(t)) + \hat{e}(b(T) - x) \qquad (13)$$

The stationarity conditions $dL/dg = 0$ and $dL/db = 0$ are

$$C'(g^T(t)) - \tilde{b}^T(t) - \hat{\lambda}^T(t) = 0 \qquad (14)$$

$$\tilde{b}^T(t) + (-\tilde{b}^T(t+1)\bar{b}^T(t) - \underline{b}^T(t))1_{t<T} + \hat{e}^T = 0 \qquad (15)$$

where C' is the derivative of C with respect to its first argument.

Making $\tilde{b}^T(t)$ the subject of equation and solving (15) gives,

$$\tilde{b}^T(t) = \sum_{\tau=t}^{T-1} \left[\underline{b}^T(\tau) - \bar{b}^T(\tau)\right] + \hat{e}^T \qquad (16)$$

By substituting (16) in (14) and applying the primal feasibility condition $g^T(t) \geq 0$, we get the optimised solution

$$C'(g^T(t);t) = \left[\sum_{\tau=t}^{T-1} (\underline{b}^T(\tau) - \bar{b}^T(\tau)) + \hat{e}^T\right]^+ \qquad (17)$$

The complementary slackness conditions for the optimal solution are,

$$(B - b^T(t))\bar{b}^T(t) = 0 \qquad (18)$$
$$b^T(t)\underline{b}^T(t) = 0 \qquad (19)$$

and

$$\hat{\lambda}^T(t)g^T(t) = 0 \qquad (20)$$

A natural consequence of these conditions is the following

Lemma 7. *Consider an any interval $[t_1, t_2]$ in which the battery is partially filled ($b^*(t) \in (0, B)$), and satisfying the technical condition that there exist $t^\dagger > t_2$ and $t^\ddagger > t_2$ such that $b^*(t^\dagger) = 0$ and $b^*(t^\ddagger) = B$.*

Then the incremental generation cost $C'(g(t);t)$ is constant on $[t_1, t_2]$. Moreover, $C'(g(t);t)$ decreases only if the battery is full $b^*(t) = B$, and increases only if the battery is empty $b^*(t) = 0$.

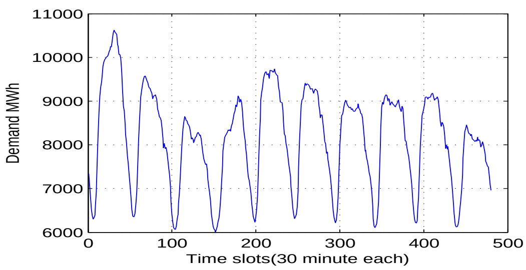

Figure 1. Daily Demand for Australian Electricity Market.

Proof: Choose $T = \max\{t^\dagger, t^\ddagger\}$ The existence of $t^\dagger$ and $t^\ddagger$ means that the hypotheses of Theorem 6 hold with $t = \min(t^\dagger, t^\ddagger)$. Thus $b^*(t) = b_x^T(t)$ on $[t_1, t_2]$ and the optimal generation g is equal in the finite and infinite horizon problems.

In the finite optimization (6)–(11), the Lagrange multipliers $\underline{b}^T$ and $\bar{b}^T$ are zero unless $b_x^T \in \{0, B\}$ by (18) and (19). Hence the first claim holds by (17). The remaining claim hold by the complementary slackness conditions and the fact that $\underline{b}^T$ and $\bar{b}$ are non-negative. ∎

Note that this is in contrast to the structure observed in [2] for finite horizon problems. That is because the model of [2] imposed a penalty for the battery being less than completely full. This penalty accumulates over slots, and so there is a greater incentive early in time to charge the battery, and less incentive nearer to the horizon.

III. Numerical Results

We now demonstrate the foregoing results numerically. The theoretical results hold for arbitrary time variations of price, $C(\cdot;\cdot)$. We numerically investigate the behaviour of the optimal solution when the cost of energy grows exponentially in real terms. We use a quadratic function of the generation following [2]; although we do not claim this to be an accurate model, we expect that the qualitative conclusions will hold for other strictly convex costs. Specifically, we study the cost function $C(t) = g(t)^2\gamma^{t/\mathcal{T}_y}$, where $\mathcal{T}_y$ is the number of time steps in a year, and $\gamma \geq 1$ is the annual rate of price increase. We use $\gamma = 1$, 1.2 and 2, corresponding to 0%, 20% and 100% growth in prices per year.

Figure 1 shows the demand in New South Wales, as reported by the Australian Energy Market Operator[10]. The data is aggregated over 30 minute intervals for 10 days in March 2012.

We model this daily variation in demand by a sinusoidal function, $d(t) = 1 + \sin(2\pi t/\mathcal{T}_d)/k$, where $\mathcal{T}_d$ is the number of slots per day. Large value of k correspond

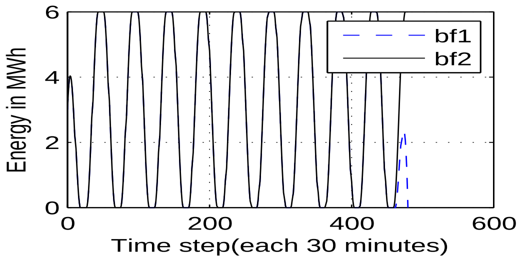

Figure 2. Renewal points for a 6MWh battery with 20% per year price increase for a 10 day horizon

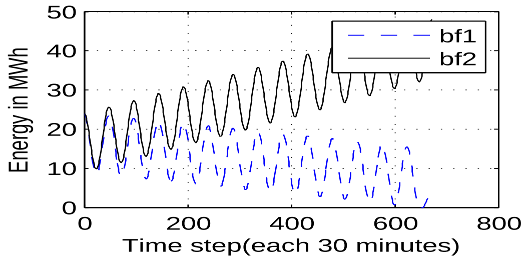

Figure 3. Charging schedule of a 48MWh battery with no increase in price per year and a 10 day time horizon

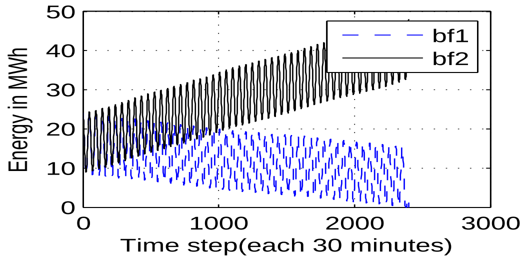

Figure 4. Charging schedule of a 48MWh battery with no increase in price per year and a 50 day time horizon

to nearly constant load, while a value of $k = 1$ is highly variable, with the peak twice the mean and some periods of zero demand. The measured data corresponds to $k \approx 3$.

The results depend substantially on the battery capacity, B. Three battery capacities used: 0.5MWh,6MWh and 48MWh which are able to store 15 minutes, 3 hours and 1 days worth of energy.

A. Renewal intervals and finite Lookahead

A central qualitative prediction is Theorem 6. It states that any interval $[t_1, t_2]$ such that either $(b_0^T(t_1), b_B^T(t_2))$ is either $(0, B)$ or $(B, 0)$ decouples the solution at times $t < t_1$ from those at times $t > t_2$, much like a renewal point does in a renewal process [11]. In particular, if the control is to be performed on-line, then the optimal strategy up to time t_1 can be calculated based on predictions of load and price only up until time t_2.

Note that it this "renewal" does not occur for every point such that $b_x^T(t) = 0$ or $b_x^T(t) = B$ for x that is neither 0 nor B. For example, if the price at $T+1$ is very high, then it is possible that $b_x^{T+1}(t) > 0$ even though $b_x^T(t) = 0$. For this reason, we refer to $[t_1, t_2]$ as a "renewal interval" instead of a renewal point.

Figure 2 shows the existence of a renewal interval for a battery $b_0(.)$ saturating above or $b_B(.)$ saturating below. For b_0, the last renewal interval is about from time step 430 to 450, whereas for b_B, the last renewal interval occurs later, from about time step 460 to 470. As stated in Theorem 6, $b_0(t) = b_B(t)$ for any t before the start of each of those renewal intervals.

Unfortunately, it is not always the case that the optimal solution b^* can be determined by looking a finite time into the future. Figure 3 shows a case where there is no interval, either for b_0^T or b_B^T, such that the battery is fully charged at one end and fully discharged at the other. This figure uses $T = 10\mathcal{T}_d$, representing 10 days. This does not mean that b^* in (12) is not well defined in this case; even if b_0^T and b_B^T do not converge for a finite horizon T, they may converge as T grows. Figure 4 shows the same system for a larger finite horizon $T = 50\mathcal{T}_d$, or 50 days. It can be seen that for a given t, such as $t = 400$, the curves $b_0^{50\mathcal{T}_d}(t)$ and $b_B^{50\mathcal{T}_d}(t)$ are much closer than $b_0^{10\mathcal{T}_d}(t)$ and $b_B^{10\mathcal{T}_d}(t)$ are.

B. Battery size, demand and price increases

Now that we have seen that renewal intervals make b^* a meaningful solution concept, let us consider how that solution varies with the battery sizes, demand fluctuation and rate of increase of prices.

Tables I to III list the fraction of time that a battery is completely full or completely empty for increasing prices and capacities. A value of "—" means that no renewal intervals exist and so convergence does not occur for finite horizons.

Unsurprisingly, these show that a reduction in fluctuation of demand causes a reduction in fluctuation of battery occupancy, and that larger batteries spend less time either totally full or totally empty.

The interesting result is that, for large 48kWh batteries, when the fluctuations in demand are small, the rate of increase makes a substantial difference in the fraction of time spent saturated.

Figure 5 shows the battery occupancy for a case in which the battery never empties. For even larger values

	time full ($b^* = B$)			time empty ($b^* = 0$)		
$k \setminus \gamma - 1$	0%	20%	100%	0%	20%	100%
1	0.38	0.38	0.38	0.38	0.38	0.38
3	0.29	0.29	0.29	0.29	0.29	0.29
20	0.08	0.08	0.08	0.08	0.08	0.08

Table I
FRACTIONS OF TIME $b^*(t) = B$ AND $b^*(t) = 0$ FOR $B = 0.5$ MWH.

	time full ($b^* = B$)			time empty ($b^* = 0$)		
$k \setminus \gamma - 1$	0%	20%	100%	0%	20%	100%
1	0.17	0.17	0.17	0.17	0.17	0.17
3	-	0.04	0.04	-	0	0
20	-	0.04	0.08	-	0	0

Table II
FRACTIONS OF TIME $b^*(t) = B$ AND $b^*(t) = 0$ FOR $B = 6$ KWH.

of γ, the battery can remain fully charged for the entire time. We can calculate how large γ must be for this to happen by noting that if the battery is always fully charged, then $g(t) = d(t)$. Charge will never be released if the price of serving $d(t)$ is monotonic increasing. That is, if

$$\begin{aligned} 0 < \frac{dC(d(t);t)}{dt} &= \frac{(d(t))^2\gamma^{t/\mathcal{T}_y}}{dt} \\ &= 2d(t)d'(t)\gamma^t + (d(t))^2\log(\gamma)\gamma^t/\mathcal{T}_y \end{aligned}$$

or equivalently if

$$\frac{\log(\gamma)}{2\mathcal{T}_y} \geq \min_t \frac{d'(t)}{d(t)}$$

For $d(t) = 1 + \sin(2\pi t/\mathcal{T}_d)/k$,

$$\min_t \frac{d'(t)}{d(t)} = \frac{2\pi}{\mathcal{T}_d}\min_\theta - \frac{\cos\theta}{k+\sin\theta}$$

This occurs when $k\sin\theta = -1$, and gives a sufficient condition of

$$\frac{\log(\gamma)}{2\mathcal{T}_y} \geq \frac{1}{\sqrt{k^2-1}}. \tag{21}$$

	time full ($b^* = B$)			time empty ($b^* = 0$)		
$k \setminus \gamma - 1$	0%	20%	100%	0%	20%	100%
1	-	0.02	0.03	-	0	0
3	-	0.05	0.06	-	0	0
20	-	0.62	0.76	-	0	0

Table III
FRACTIONS OF TIME $b^*(t) = B$ AND $b^*(t) = 0$ FOR $B = 48$ KWH.

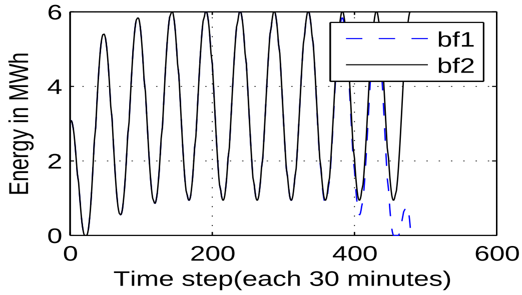

Figure 5. The charging schedule for a 6MWh battery with price increasing by a factor of 5.76 per year

To understand the implications of this, note that when the battery is always full, its only role is to hedge against rising prices, and its role of peak shaving disappears. This shows that, as demand fluctuations become small (large k), the battery will cease peak shaving for moderately small rates of price growth, γ. Conversely, as the minimum demand tends to 0 (k tends to 1), peak shaving will remain useful even for arbitrarily large rates of price growth. This latter phenomenon is a consequence of the fact that the incremental price is zero when $g(t) = 0$; if the incremental cost remains non-zero then even if the demand drops to zero then extremely high price growth will inhibit peak shaving.

The degree to which peak shaving is inhibited can be measured in terms of the maximum generation rate. With perfect peak shaving, the maximum generation will equal the mean demand, which is is 1 kWh per 30 minute slot, whereas with no peak shaving the maximum generation will equal the peak demand. This is shown in Figure6. A price increase of 10^{-5} per half hour corresponds to a growth in price of 20% per year, which is a plausible value as energy becomes scarcer. The figure shows that the battery will remain effective at peak shaving for such price rises. However, if the rate of price increase is extreme, as at the right of the graph, then infrastructure must be dimensioned to carry the peak demand, regardless of the size of users' batteries. These prices rises, of many percentage points per month, are not likely to be sustained, but may easily be induced by market volatility. Even if such rises occur seldom, the grid may need to be dimensioned to accommodate them. This will have a substantial impact on the amount to which user-controlled storage, such as vehicle-to-grid systems, can be relied on for load smoothing.

These results suggest that, when demand fluctuation is significant, a moderate rate of price increase has minimal impact. Actually, there is one marked qualitative effect

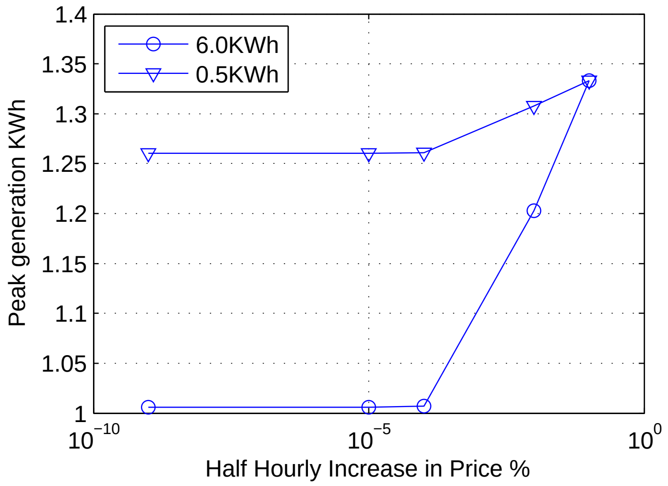

Figure 6. Peak generation during convergence vs Rate of increase for a 0.5MWh,6MWh and 48 MWh battery and a 50 days time horizon

of the long-term trend in price in the current model. The traditional approach to infinite horizon problems, namely discounting future costs, assumes the long-term trend is that prices decrease in real terms. In this case, it is optimal for a large battery to maintain a low charge, only sufficient to cover peak shaving. In contrast, when prices are rising — however slowly — it is optimal for the battery to maintain a high charge. However, this conclusion depends on two important simplifications in the model in Section II, as follows.

The first is that batteries do not leak energy. In practice, it may not be feasible to store energy for prolonged periods, which reduces the incentive to use small-scale storage to combat increasing prices. Particularly, flywheels are only able to store energy for times on the order of a day. Chemical batteries leak energy over periods of years, while dams leak energy through evaporation. Although this leakage may be slow, it must be compared against the slow rate of price increase.

The other simplification is that there is no intrinsic cost associated with the battery's state of charge. In reality, wear-and-tear costs on batteries depend on their state of charge; for example, lead-acid batteries wear out rapidly if stored at low charge, while lithium-ion batteries wear out faster if stored at high charge. Again, these considerations may dominate a slow rate of price increase, and should be incorporated into future models.

IV. Conclusion

When the price of fuel is set to rise, motorists often "panic buy" and stockpile fuel. We have investigated the question of whether such panic buying is likely to be useful in electric vehicles, and the possible impact it would have on vehicle-to-grid peak shaving systems. We show that long-term price rises in the range expected to result from the natural increase in scarcity of energy should not be sufficient to cause small batteries to be used for long-term storage. However, short term fluctuations in which prices rise by several percentage points per month may have such an impact. This limits the amount to which user-controlled peak-shaving can be relied on by utility companies.

Acknowledgement

This work was supported in part by the Australian Research Council grant FT0991594.

References

[1] B. Daryanian, R. Bohn, and R. Tabors, "Optimal demand-side response to electricity spot prices for storage-type customers," *Power Systems, IEEE Transactions on*, vol. 4, no. 3, pp. 897 –903, aug 1989.

[2] K. Chandy, S. Low, U. Topcu, and H. Xu, "A simple optimal power flow model with energy storage," in *Proc. 49th IEEE Conference on Decision and Control (CDC), 2010*, dec. 2010, pp. 1051 –1057.

[3] P. Harsha and M. A. Dahleh, "Optimal sizing of energy storage for efficient integration of renewable energy," in *Proc. 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), 2011*, Orlando, FL, USA, Dec 2011.

[4] M. R. A. Faghih and M. A. Dahleh, "Optimal utilization of storage and the induced price elasticity of demand in the presence of ramp constraints," in *Proc. 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), 2011*, Orlando, FL, USA, Dec 2011.

[5] P. Van de Ven, N. Hegde, L. Massoulie, and T. Salonidis, "Optimal control of residential energy storage under price fluctuations," in *Proc. IARIA ENERGY, May 2011*, Venice Italy, May 2011.

[6] M. Korpaas, A. T. Holen, and R. Hildrum, "Operation and sizing of energy storage for wind power plants in a market system," *International Journal of Electrical Power & Energy Systems*, vol. 25, no. 8, pp. 599 – 606, 2003. [Online]. Available: `http://www.sciencedirect.com/science/article/pii/S0142061503000164`

[7] E. Bitar, A. Giani, R. Rajagopal, D. Varagnolo, P. Khargonekar, K. Poolla, and P. Varaiya, "Optimal contracts for wind power producers in electricity markets," in *Proc. 49th IEEE Conference on Decision and Control, 2010*, Atlanta, GA, USA, Dec 2010.

[8] I. Koutsopoulos, V. Hatzi, and L. Tassiulas, "Optimal energy storage control policies for the smart power grid," in *Smart Grid Communications (SmartGridComm), 2011 IEEE International Conference on*, oct. 2011.

[9] Y. Dirickx, "Deterministic discrete dynamic programming with discount greater than one: Structure of optimal policies," *Management Science*, pp. 32–43, 1973.

[10] A. E. M. Operator, http://www.aemo.com.au/Electricity/NEM-Data/Price-and-Demand-Data-Sets, [Online].

[11] S. Ross, *Introduction to probability models*. Academic press, 2009.

APPENDIX

Lemma 8. *If $x < y$ and there is no $t \in [\tau, T]$ such that $b_x^T(t) = b_y^T(t)$, then $b_x^T(t) < b_y^T(t)$ for all $t \in [\tau, T]$.*

Proof: Since there is no t such that $b_x^T(t) = b_y^T(t)$, then the only way for Lemma 2 to be false is if there is a t for which $b_x^T(t) < b_y^T(t)$ and $b_x^T(t-1) > b_y^T(t-1)$. We will now show that this leads to a contradiction.

Let $\mathcal{B}_x(t-1)$, and $\mathcal{B}_y(t-1)$ be the optimal cost for b_x^T and b_y^T respectively from time $[\tau, t-1]$. Note that the costs from $[t-1, t]$ for b_x^T and b_y^T are $C(g_x^T(t); t)$ and $C(g_y^T(t); t)$ respectively. Let

$$g_x^T(t) = \Theta = b_x^T(t) - b_x^T(t-1) + d(t) \tag{22}$$

$$g_y^T(t) = \Phi = b_y^T(t) - b_y^T(t-1) + d(t) \tag{23}$$

$$\theta = b_y^T(t) - b_x^T(t-1) + d(t) \tag{24}$$

$$\phi = b_x^T(t) - b_y^T(t-1) + d(t). \tag{25}$$

Note that $b_x^T(t-1) > b_y^T(t-1)$ and $b_x^T(t) < b_y^T(t)$ would imply $\Theta < \min(\theta, \phi)$ and $\Phi > \max(\theta, \phi)$. Moreover $\Theta + \Phi = \theta + \phi$. Hence there is a $\delta \in (0, 1)$ such that $\theta = (1-\delta)\Theta + \delta\Phi$ and $\phi = \delta\Theta + (1-\delta)\Phi$. The convexity of C then implies

$$C(\theta; t) + C(\phi; t) < C(\Theta; t) + C(\phi; t). \tag{26}$$

Then by adding the costs $\mathcal{B}_x(t-1) + \mathcal{B}_y(t-1)$ to equation (26) we get,

$$\begin{aligned} &\mathcal{B}_y(t-1) + C(\phi; t) + \mathcal{B}_x(t-1) + C(\theta; t) < \\ &\mathcal{B}_x(t-1) + C(\Theta; t) + \mathcal{B}_y(t-1) + C(\Phi; t) \end{aligned} \tag{27}$$

Equation (27) shows that, either

$$\mathcal{B}_x(t-1) + C(\Theta; t) > \mathcal{B}_x(t-1) + C(\theta; t) \tag{28}$$

which means that it is strictly better for b_x^T to follow the path of b_y^T after time $t-1$ contradicting the optimality of b_x^T, or

$$\mathcal{B}_y(t-1) + C(\Phi; t) > \mathcal{B}_y(t-1) + C(\phi; t) \tag{29}$$

which means that it is strictly better for b_y^T to follow the path of b_x^T after time $t-1$ contradicting the optimality of b_y^T.

Hence it is not possible for both $b_x^T(t) < b_y^T(t)$ and $b_x^T(t-1) > b_y^T(t-1)$, which establishes the result. ■