Collision Probability in Saturated IEEE 802.11 Networks

Hai L. Vu

Centre for Advanced Internet Architectures (CAIA)
ICT Faculty, Swinburne University of Technology
Hawthorn, VIC 3122, Australia
h.vu@ieee.org

Taka Sakurai

ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN)
E&E Eng. Department, The University of Melbourne
Melbourne, VIC 3010, Australia
tsakurai@ieee.org

Abstract

In this paper we study the impact of channel capture following a busy period on collision probabilities in a saturated IEEE 802.11 network. A new analytical model that takes into account the previous channel status is proposed to obtain collision probabilities. Using simulation, we show that the well-known fixed point analysis, which is a special case of the new model, becomes more accurate as the size of the initial backoff window increases.

1. Introduction

Wireless local area networks (WLANs) based on the IEEE 802.11 standard [1] have been widely deployed. The default medium access control (MAC) protocol for channel access in this standard is the distributed coordination function (DCF), which is a random access scheme based on carrier sense multiple access with collision avoidance (CSMA/CA). In this paper we consider a WLAN using DCF in a so-called infrastructure network, where every station in the network communicates via the access point (AP). Furthermore, we assume that stations always have packets to sent, i.e. they are saturated.

2. IEEE 802.11 DCF Protocol Description

In this section we briefly describe the DCF mechanism as detailed in the IEEE 802.11 standard [1]. According to the DCF mechanism, whenever a station has a packet to send it should defer its transmission for a guard period known as the distributed interframe spacing time (DIFS), during which the channel must be sensed idle. This is followed by a random backoff interval. Backoff intervals are slotted, and stations are only permitted to commence their transmissions at the beginning of slots. When backoff is initiated, a random integer backoff time is selected from the range $[0; CW - 1]$ with a uniform distribution, where CW is a contention window. At the first transmission attempt, CW is set equal to W, the minimum contention window. The backoff time counter is decremented as long as the channel is sensed idle. It is frozen when activities (i.e. packet transmissions) are detected on the channel, and reactivated after the channel is sensed idle again for a guard period. This guard period is equal to a DIFS if the transmitted packet was error-free, and equal to the extended interframe spacing time, EIFS, if there was a collision. The station transmits its packet when the backoff time counter reaches zero. A collision occurs when the counters of two or more stations reach zero in the same slot.

If the packet transmission is successful, the receiving MAC layer sends an ACK after a short interframe spacing time, SIFS. As SIFS < DIFS the ACK will reach the transmitting node without the need to contend for the channel. If the packet transmission is unsuccessful (indicated by an ACK timeout), the size of the contention window is doubled, and another backoff period is initiated. This doubling continues until CW has reached its maximum value, $CWmax = 2^m W$, where m is the number of window doublings allowed. After m unsuccessful attempts, the window is maintained at $CWmax$ for the remaining attempts until the packet is successful, or until the maximum number of attempts (K) is reached, after which the packet is discarded.

3. Collision Probability: a Fixed Point Analysis

Collision probability has been extensively studied as it is a vital ingredient for any performance evaluation (such as throughput or delay evaluation) of the IEEE 802.11 system. In [4] the author provided a Markov chain model, which was then simplified and further developed in [9]. The latter paper shows that a simple mean value analysis is enough to obtain accurate predictions of collision probability. The author in [4] introduced a key approximation that enabled the estimation of p, the probability of collision seen by a packet being transmitted on the channel. This approximation is based on the assumption that each packet collides with constant and independent probability p, and is also independent of the channel status. The channel independency, however, does introduce some inaccuracy as we will describe in the next section. This approximation leads to a fixed point formulation for p which is summarised below.

Recall that the contention window is initially set to W. If p is the collision probability, then an arbitrary packet is successfully transmitted with probability $1 - p$, and the average backoff window of such a packet is $(W - 1)/2$. If the first transmission fails, the packet is successfully transmitted on the second attempt with probability $p(1 - p)$. The average backoff window in this case is $(2W - 1)/2$. This argument can be continued up to the last (Kth) permitted transmission. The backoff window, however, will only be increased until it reaches the $CWmax$ value.

The overall average backoff window can be calculated from

$$
\begin{aligned}
W_{avg} = \ & \eta(\frac{W - 1}{2}) \qquad\qquad (1) \\
+ \ & \eta p(\frac{2W - 1}{2}) + ... \\
+ \ & \eta p^m(\frac{2^m W - 1}{2}) \\
+ \ & \eta p^{m+1}(\frac{2^m W - 1}{2}) + ... \\
+ \ & \eta p^{K-1}(\frac{2^m W - 1}{2}),
\end{aligned}
$$

where

$$
\eta = \frac{1 - p}{1 - p^K},
$$

and $(1 - p^K)$ is a normalisation term to ensure the probability of each backoff stage follows a valid probability distribution. After some algebraic manipulations we obtain

$$
\begin{aligned}
W_{avg} = \ & \frac{1}{1 - p^K} \qquad\qquad\qquad (2) \\
\times \ & \left(\frac{W(1 - p)(1 - (2p)^m)}{2(1 - 2p)} - \frac{1 - p^m}{2} \right. \\
+ \ & \left. \frac{(2^m W - 1)(p^m - p^K)}{2} \right).
\end{aligned}
$$

Based on an overall average backoff window, the probability that a station attempts to transmit in an arbitrary slot is given by $1/W_{avg}$. The probability that during the transmission of an arbitrary station there is no other active station is $(1 - 1/W_{avg})^{n-1}$. The collision probability p is then given by

$$
p = 1 - (1 - 1/W_{avg})^{n-1}. \qquad (3)
$$

Equations (2) and (3) establish a fixed point formulation from which the collision probability p can be computed using a numerical technique.

It can be shown that the above fixed point equations have a unique solution, and is insensitive to the distribution of the backoff time. The conditions and uniqueness properties are discussed in detail in [8].

4. Collision Probability: State Dependent Analysis

As described in Section 2, a DCF station can only access the channel and commence transmission when its backoff time counter reaches zero. During the counting down process, if the channel becomes busy (i.e. there is a transmission on the channel), stations with non-zero backoff time will stop their backoff counter, and only resume it when the channel becomes idle again after a guard period.

As a station reactivates its backoff counter, it is clear that the station will not be able to access the channel during the immediate time slot after the guard time of the busy period. Due to this reason, the probability that a station accesses the channel depends on whether the channel was idle or busy in the previous time slot. In particular, after an idle slot, any station can access the channel as long as their backoff time counter is zero. On the other hand, after a busy period, if there was a collision then only those stations involved in the collision may access the channel if they happened to choose zero as their new backoff period. If there was a successful transmission then only the station who just accessed the channel may access it again if its new chosen backoff is zero. We identify this effect as a channel capture effect, and refer to the time slot immediately following the DIFS guard time after a successful transmission as a *post-DIFS slot*. The effect on channel access probability caused by a post-DIFS slot after a collision is the same as in the case of successful transmission, however, the guard time in this case is EIFS.

The authors in [5] proposed an analytical model taking into account this post-DIFS effect by extending the Markovian model developed in [4]. Their results did show some impact of the post-DIFS slot. However, the obtained numerical results for collision probability seem too low for such a saturated WLAN network. Below we present a new analytical model based on a simpler mean value approach that

utilises the channel access probability conditioned upon the status of the channel from the previous time slot.

Let us again consider n stations, each operating in saturation. As explained in the previous section, the average backoff window can be expressed as a function of the collision probability p and is given in (2). And the probability τ that a station attempts to transmit (or to access channel) in an arbitrary slot is given by $1/W_{avg}$.

Hence the probability that a slot is idle (P_i), i.e. no station transmits, is given by:

$$P_i = (1 - 1/W_{avg})^n. \qquad (4)$$

Let τ_i and τ_b be the probabilities that a station accesses the channel after an idle and busy slot, respectively. The probability that a station attempts to transmit in an arbitrary slot can be expressed as

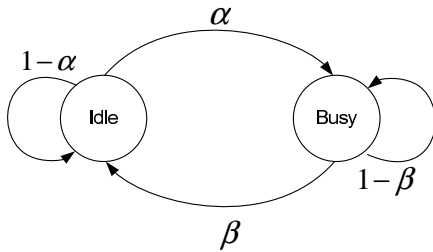$$\tau = P_i \tau_i + (1 - P_i)\tau_b = 1/W_{avg}. \qquad (5)$$

Figure 1. Markov chain model for channel idle-busy periods.

We use a two-state Markov chain in Fig. 1 to model the different types of time periods (busy, idle) on the channel. Let α be the probability that the channel becomes busy given that it was idle in the previous slot. And similarly, let β be the probability that the channel is idle in the current slot given that it was busy in the previous one. Using the conditional channel access probabilities τ_i and τ_b we can compute the probability that the channel remains idle after an idle slot as: $1 - \alpha = (1 - \tau_i)^n$, and the probability that channel becomes idle after a busy period as: $\beta = (1 - \tau_b)^n$.

The stationary probability of an idle state of the above Markov chain is then given by:

$$P_i = \frac{\beta}{\alpha + \beta} = \frac{(1 - \tau_b)^n}{1 - (1 - \tau_i)^n + (1 - \tau_b)^n}. \qquad (6)$$

Given that the previous slot is idle, the probability that during the transmission of an arbitrary station there is no other active station is $(1 - \tau_i)^{n-1}$. Thus the collision probability in this case is $P_i[1 - (1 - \tau_i)^{N-1}]$. In general, the collision probability can be expressed as:

$$p = P_i[1 - (1 - \tau_i)^{n-1}] + (1 - P_i)[1 - (1 - \tau_b)^{n-1}]. \qquad (7)$$

Equations (2), (4), (5), (6) and (7) establish a fixed point formulation from which the collision probability p, and the conditional channel access probabilities τ_i and τ_b can be computed.

Note that one trivial root of the above fixed point formulation can be obtained when $\tau_i = \tau_b = \tau$, i.e. the channel access probability is the same regardless of the status of the previous time slot. In this case the new fixed point formulation simplifies to (3), which has an unique solution. In general, however, the new fixed point formulation has many solutions and it is difficult to obtain a desired solution which reflects the effect of the post-DIFS slot. For this reason, in the next section we will show what impact the post-DIFS slot has on the collision probability by mean of simulation.
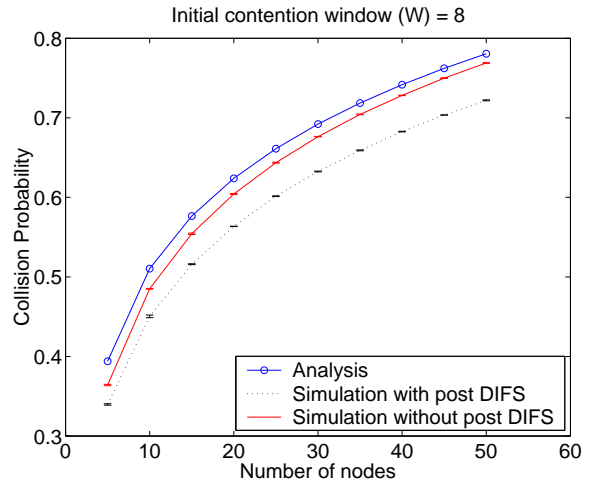

Figure 2. Simulation and analytical results for collision probability using $W = 8$.

5. Simulation Results

The simulation was performed using the ns-2 simulator [11] (version 2.27) which has a built-in implementation of the IEEE 802.11 MAC. The network scenario that we simulate is a population of n saturated stations sending packets to an access point (AP), in ideal channel conditions. The stations use the UDP protocol with a fixed packet size of 33 bytes. We choose MAC and physical layer parameter values consistent with an 802.11b system [2]. The simulation results obtained from ns-2 are plotted with 95% confidence intervals with 5 runs for each point.

The collision probability is measured in the above saturated network using different initial contention windows. These values of minimum contention window are chosen to be 8, 16 and 32. We compare the simulation results with
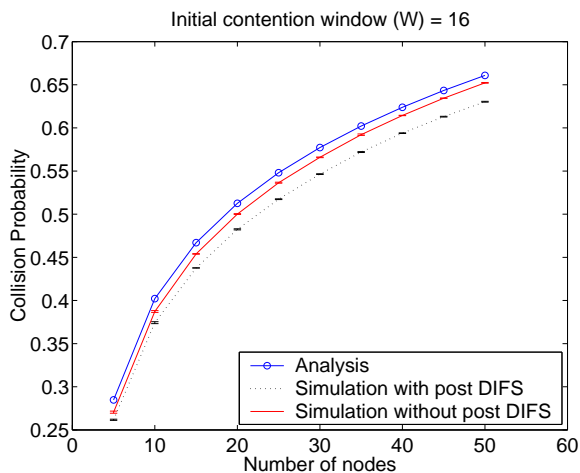
Figure 3. Simulation and analytical results for collision probability using $W = 16$.

Figure 4. Simulation and analytical results for collision probability using $W = 32$.

the collision probabilities obtained from our channel state dependent analytical model. As mentioned in the previous section, solving the fixed point formulation developed from the state dependent model is difficult since the solution is not unique. Here we choose to solve the fixed point equations for the special case when the conditional channel access probabilities are equal, i.e. $\tau_i = \tau_b$. In this case the fixed point problem is the same as the one presented in Section 3 which has a unique solution. The analytical results obtained in this case, in fact, do not reflect the post-DIFS impact on collision probability as the conditional channel access probability is the same in the time slot immediately after an idle or busy channel periods.

Collision probabilities obtained from simulation and the analytical model are shown in Figs 2, 3 and 4 for the values of minimum contention window 8, 16 and 32, respectively. Observe that there is a difference between simulation and analytical results which is expected since the analytical results are obtained for a special case. The gap between simulation and analytical results, however, decreases as the initial contention window increases. This can be explained by the fact that the impact of post-DIFS slots decreases as the contention window gets larger. Specifically, the probability that a station that successfully transmitted or was involved in the collision in the previous busy period, will attempt to access the channel in the post-DIFS slot decreases. Results presented in Fig. 4 are consistent with results reported in [8], although the authors in [8] did not explain why there was the gap between their simulation and analytical results.

To confirm the impact caused by the post-DIFS slots we have modified the simulation so that all stations have a chance to access the post-DIFS slot, rather than only those
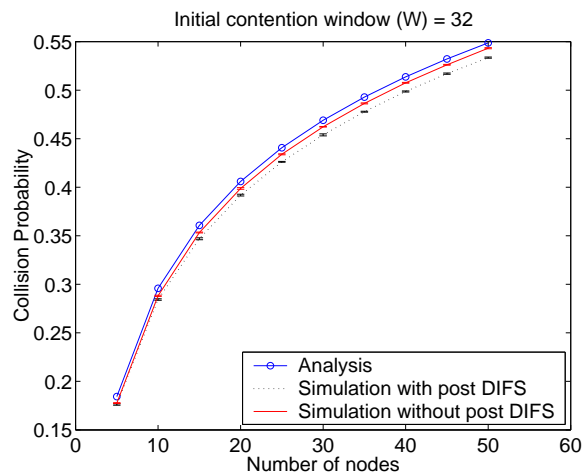
stations who were active in the previous busy period. The new simulation results are shown in the same figures for comparison purposes. Our results show that the analytical results match well with new simulation results obtained by eliminating the post-DIFS effect for all the cases.

Note that in the new IEEE 802.11e standard [3], the mechanism of backoff timer counter is changed such that all stations have a chance to access the post-DIFS slot, and thus the fixed point formulation previously proposed in [9] will give accurate results as confirmed via simulation in this section.

6. Conclusion

In this paper we have investigated the impact of post-DIFS slots on collision probabilities in a saturated 802.11 WLAN. We have proposed a new channel state dependent model to obtain collision probabilities that takes into account the previous channel status. We show via simulation that the effect of post-DIFS slots is decreasing as the size of the initial contention window increases. Furthermore, we show that the previously proposed simple analytical model, which is a special case of the new model proposed in this paper, gives accurate results when the post-DIFS slots are eliminated.

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